Direct product

In group theory, the direct product of two groups \((G, *)\) and \((H, o)\), denoted by \(G \times H\) is the set of the elements obtained by taking the cartesian product of the sets of elements of \(G\) and \(H\): \(\{(g, h) : g \text{ in } G, h \text{ in } H\}\).

For abelian groups which are written additively, it may also be called the direct sum of two groups, denoted by 
\[ G \oplus H \]

The group obtained in this way has a normal subgroup isomorphic to \(G\) (given by the elements of the form \((g, 1)\)), and one isomorphic to \(H\) (comprising the elements \((1, h)\)).

The reverse also holds: if a group \(K\) contains two normal subgroups \(G\) and \(H\), such that \(K = GH\) and the intersection of \(G\) and \(H\) contains only the identity, then \(K = G \times H\). A relaxation of these conditions gives the semidirect product.

Contributors

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